Proportion

BUSINESS MATH

ABM STRAND

Lesson Objectives

- write proportions illustrating real-life situations;
- define and give examples of a proportion;
- use the fundamental property of proportions to show if two ratios form a proportion and to solve a proportion;
- solve a word problem that may be translated as a proportion;
- recognize if a proportion is direct, is an inverse proportion, or is partitive;
- solve problems involving direct proportions, inverse proportions, and partitive proportions.

Motivation (Focusing Event)

Carl received cash from his relatives on his birthday. He got EUR 20 from his mother, USD 12 from his sister, and JPY 2000 from his father. He was told that USD 2 is equivalent to PhP92.20, USD 10 is equivalent to EUR 10, and PhP1 is equivalent to JPY2.34. How much money does he have in pesos?

Proportion

DEFINITION

A statement that two ratios are equal is called a **proportion**.
If $\frac{a}{b}$ and $\frac{c}{d}$ are two equal ratios, then the statement $\frac{a}{b} = \frac{c}{d}$ is called a **proportion**.

Each of the four numbers in a proportion is called a **term** of the proportion.

From $\frac{a}{b} = \frac{c}{d}$, **a** is the first term; **b** the second term; **c** the third term; and **d** the fourth term. The first and fourth terms are called the **extremes**. The second and third terms are called the **means**.

Example 1:

In the proportion $\frac{5}{6} = \frac{10}{12}$, name the four terms, the means, and the extremes.

Solution: First term = 5 Second term = 6 Third term = 10 Fourth term = 12
The means are 6 and 10; the extremes are 5 and 12.

Fundamental Property of Proportions

In any proportion, the *product of the means* is equal to the *product of the extremes*. That is, the cross products of the terms are equal. In symbols,

if $\frac{a}{b} = \frac{c}{d}$, then ad = bc.

Example 2:

We use the Fundamental Property of Proportions to verify that $\frac{7}{8} = \frac{14}{16}$. Equating the cross products of the terms gives
7 x 16 = 8 x 14. That is, 112 = 112.

Example 3:

Do the ratios $\frac{8}{10}$ *and* $\frac{18}{22}$ form a proportion? Explain.

Solution:

We compute for the cross products of $\frac{8}{10}$ *and* $\frac{18}{22}$. If they are equal, then $\frac{8}{10}$ *and* $\frac{18}{22}$ form a proportion.
We have 8 x 22 = 176; while 10 x 18 = 180. Since $176 \neq 180$, $\frac{8}{10}$ *and* $\frac{18}{22}$ do not form a proportion.

Example 4:

We also use the Fundamental Property of Proportions to find the missing term in a proportion as shown below.

Given $\frac{7}{8} = \frac{n}{16}$. We set cross products equal: 8n = 7(16) or n = 14.

Example 5:

Solve for n: $\frac{n+4}{5} = \frac{n-2}{3}$.

Solution: The cross products are equated as shown below.

$$5(n - 2) = 3(n + 4)$$
$$5n - 10 = 3n + 12$$
$$2n = 22$$
$$n = 11$$

Check: Verify that 11 is the solution.

MOTIVATION (FOCUSING EVENT)

Carlo, Danie, and Mia are going to share PhP18,000. Carlo will receive $\frac{1}{2}$ of what Mia's going to receive, while Danie will get $\frac{3}{4}$ of what Mia is going to get. How much will each one receive?

Example 1:

Eight tea bags are needed to make 5 liters of iced tea. How many tea bags are needed to make 15 liters of iced tea?

Solution:

Let t represent the number of tea bags. The problem translates to the following proportion:

$$\frac{t}{15} = \frac{8}{5}$$

Note that both ratios in the proportion compare the same quantities; that is, both ratios compare number of tea bags to liters of iced tea. In words, the proportion says "t tea bags to 15 liters of iced tea as 8 tea bags to 5 liters of iced tea".

Using the fundamental property of proportions, we obtain the following:

$\frac{t}{15} = \frac{8}{5} \rightarrow 5(t) = 15(8)$ t = 24 tea bags

Example 2:

A manufacturer knows that during an average production run, out of 1,000 items produced by a certain machine, 25 will be defective. If the machine produces 2,030 items, how many can be expected to be defective?

Solution:

We let x represent the number of defective items and solve the following proportion:

$$\frac{x}{2030} = \frac{25}{1000}$$

Example 3:

If 1 out of 6 people buy a particular branded item, how many people can be expected to buy this item in a community of 6,000 people?

Solution:

Let p = the number of people buying the branded item. The ratio $\frac{p}{6000}$ defines the number of people p out of 6000 buying the branded item. This ratio is equal to 1 to 6. These two ratios are equal; that is, they form a proportion as given below.

$$\frac{p}{6000} = \frac{1}{6}$$

Solving for p, we get p = 1000. So, 1000 people can be expected to buy the particular branded item.

How do we recognize whether a given proportion problem involves a:

- ✓ direct proportion
- ✓ an inverse proportion, or
- ✓ a partitive proportion?

Types of proportion (variation):

1. **Direct proportion** – two variables, say x and y, varying such that as x increases, y also increases or as x decreases, y also decreases proportionally; that is, the ratio $\frac{x}{y}$ is always the same. The same holds true with the ratio $\frac{y}{x}$.
2. **Indirect/Inverse** – two variables, say x and y, varying such that as x increases, y decreases, or as x decreases, y increases proportionally; that is, the product of x and y is always the same.
3. **Partitive proportion** – a whole is divided into more than two parts.

Let us work on these:

Example 4:
Two boxes of chocolates cost PhP180. How much do 7 boxes of chocolates cost?

Example 5:
Forty liters of water is transferred into 3 containers in the ratio 1:3:4. How much water is in each
container?

Example 6:
If Trina works 20 hours, she earns PhP600. How much does she earn if she works 30 hours?

Example 4:

Two boxes of chocolates cost PhP180. How much do 7 boxes of chocolates cost?

<u>Solution</u>: The more the boxes, the higher the cost; that is, both quantities are increasing. We have a direct proportion. The ratio $\frac{\text{no. of box}}{\text{cost}}$ is always the same. That is, $\frac{\text{no. of box}}{\text{cost}} = \frac{\text{no. of box}}{\text{cost}}$ Substituting the given values, we have

$$\frac{2}{180} = \frac{7}{\text{cost}} \quad \text{cost} = \frac{180 \times 7}{2} = \text{P } 630$$

Example 5:

Forty liters of water is transferred into 3 containers in the ratio 1:3:4. How much water is in each container?

<u>Solution:</u>

The ratio 1:3:4 indicates 1 + 3 + 4 = 8 portions. 40 liters will be divided into 8 portions; that is, $\frac{40}{8}$ = 5 liters (L) per portion.

Container 1 (1 portion) = 1 x 5 L = 5 L in it.
Container 2 (3 portions) = 3 x 5 L = 15 L in it.
Container 3 (4 portions) = 4 x 5 L = 20 L in it.

Example 6:

If Trina works 20 hours, she earns PhP600. How much does she earn if she works 30 hours?

Solution:

This is a direct proportion problem; that is, the more hours Trina works, the more she earns. Let x represent Trina's earnings for working 30 hours. The ratio is the always the same. That is,

$$\frac{20 \text{ hours}}{600 \text{ pesos}} = \frac{30 \text{ hours}}{x \text{ pesos}} \rightarrow x = \frac{30 \text{ hours} \times 600 \text{ pesos}}{20 \text{ hours (pesos)}} = 900$$

Mastery Test

PROPORTION

Answer the following:

1. Jessa buys three bananas for PHP25.00. How much does she have to pay for a dozen of these bananas?
2. A typist can finish 4 pages in 6 minutes. How long will it take him to finish 18 pages?
3. A menu which serves 5 people requires 3 cups of flour. How many cups of flour are needed for the menu to serve 20 people?

Answer the following:

4. To finish a certain job in 8 days, 6 workers are needed. If it is required to finish the same job in 2 days advance, how many workers have to work?
5. A supply of food lasts for a week for 20 families. How long would the supply last if 3 more families have to be supplied?
6. A deceased person stated in his testament that his 30-hectare land be divided among his three children using 1:2:3 partition, the oldest getting the biggest share. How much did the second child receive?
7. The ratio of cups of water to cups of sugar in a menu is 3:1: 1/2. If this is just for one serving, how much of each is needed for a menu that makes 5 servings?

Mastery Test

KEY TO CORRECTION

CHECKING FOR UNDERSTANDING

1. Jessa buys three bananas for PHP25.00. How much does she have to pay for a dozen of these bananas?

Solution: $\frac{3}{25.50} = \frac{12}{x} \rightarrow 3x = 12\,(25.50) \rightarrow x =$ **P 102**

2. A typist can finish 4 pages in 6 minutes. How long will it take him to finish 18 pages?

Solution: $\frac{4}{6} = \frac{18}{x} \rightarrow 4x = 6(18) \rightarrow x =$ **27 minutes**

3. A menu which serves 5 people requires 3 cups of flour. How many cups of flour are needed for the menu to serve 20 people?

Solution: $\frac{5}{3} = \frac{20}{x} \rightarrow 5x = 3(20) \rightarrow x =$ **12 cups**

4. To finish a certain job in 8 days, 6 workers are needed. If it is required to finish the same job in 2 days advance, how many workers have to work?

Solution: (8 days)(6 workers) = (6 days)(x workers) → 6x = 8(6) → x = **8 workers**

5. A supply of food lasts for a week for 20 families. How long would the supply last if 3 more families have to be supplied?

Solution: (7 days)(20 families) = (x days)(23 families) → 23x = 7(20) → x = **about 6 days**

6. A deceased person stated in his testament that his 30-hectare land be divided among his three children using 1:2:3 partition, the oldest getting the biggest share. How much did the second child receive?

Solution: $\frac{30}{1+2+3} = \frac{30}{6} = 5 \rightarrow 2(5) =$ **10**

7. The ratio of cups of water to cups of sugar in a menu is $3:1:\frac{1}{2}$.if this is just for one serving, how much of each is needed for a menu that makes 5 servings?

Solution: $5\,(3:1:\frac{1}{2}) \rightarrow$ **$15:5:\frac{5}{2}$**

For lesson handout, visit:

http://jacs.weebly.com

THANK YOU!